Shear-deformable Hybrid Finite Element Method for Buckling Analysis of Composite Thin-walled Members

By

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Thesis submitted in fulfilment
of the full requirement for the degree of
Doctor of Philosophy

School of Civil and Environmental Engineering

Faculty of Engineering and Information Technology

University of Technology Sydney (UTS)

December 2018

Certificate of authorship and originality

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

This research is supported by the Australian Government Research Training Program.

Vida Niki

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To Ashkan

Acknowledgements

I would like to express my deepest appreciation to my supervisor Dr. R. Emre Erkmen for his continuous support and guidance throughout my studies. His immense knowledge, motivation and patience helped me bring this study into success. I could not have imagined having a better supervisor for my Ph.D study.

My sincere thanks also go to Professor Bijan Samali who provided me an opportunity to do my PhD in Australia. His kind and insightful advice helped me to overcome the obstacles I have been facing throughout my studies.

I would especially like to express my heartfelt gratefulness for having my beloved husband, Dr. Ashkan Afnani, by my side who always has been a source of motivation for me.

Words cannot express how grateful I am to my parents, Mohebatollah and Behrokh, for all of their sacrifices, encouragement and support without which doing my PhD would have been impossible.

Finally, I would like to thank my lovely sons, Ryan and Nickan, who could manage to be the source of distraction and inspiration at the same time for me throughout my studies.

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List of Symbols

$\phi(z)$	=	angle of rotation of the cross-section
Δ	=	lateral deflection of flanges
$\sigma(s)$	=	normal stress
$\tau(s)$	=	shear stress
P	=	normal force
M_x	=	bending moment about x axis
M_y	=	bending moment about y axis
B	=	bimoment
Q_y	=	shear force in y direction
Q_x	=	shear force in x direction
T_v	=	Saint Venant twist
T_ω	=	flexural twist
A	=	area of the cross-section
I_x	=	moment of inertia of the cross-section around the x axis
I_y	=	moment of inertia of the cross-section around the y axis

$\omega(s)$	=	sectorial coordinate of the cross-section
$S_\omega(s)$	=	sectorial moment of the cross-section
I_ω	=	sectorial moment of inertia of the cross-section
$S_\omega(s)$	=	sectorial moment of area
$x(s), y(s)$	=	coordinates of an arbitrary point P on the mid-surface
a_x, a_y	=	coordinates of a pole A on the cross-section
$u(s,z)$	=	horizontal displacement of point P
$v(s,z)$	=	vertical displacement of point P
$w(s,z)$	=	longitudinal displacement of point P
γ_{zt}	=	shear strain on the mid-surface
t	=	tangential component of displacement at the mid-surface
α	=	angel between the tangent at point P and the x axis
P_{cr}	=	critical buckling load
xyz	=	local coordinate at the pre-buckling state
$x'y'z'$	=	local coordinate at the buckled state
E	=	Modulus of Elasticity of the material
G	=	shear modulus

J	=	torsional constant of the cross-section
C_w	=	cross-sectional warping constant
M_{cr}	=	critical buckling moment
u_T	=	lateral displacement of the top flange
u_B	=	lateral displacement of the bottom flange
ϕ_T	=	rotation of the top flange
ϕ_B	=	rotation of the bottom flange
Φ_k	=	fibre orientation of k^{th} layer of composite laminate cross-section
σ_1, σ_2	=	Stresses in two directions for orthotropic material
$\varepsilon_1, \varepsilon_2$	=	Strains in two directions for orthotropic material
E_1, E_2	=	Young's moduli in two directions for orthotropic material
ν_{12}, ν_{21}	=	Poisson's ratios in two directions for orthotropic material
τ_{12}	=	shear stress
γ_{12}	=	shear strain
$\boldsymbol{\sigma}$	=	stress vector
$\boldsymbol{\varepsilon}$	=	strain vector

$\mathbf{Q}$	=	constitutive matrix for composite material
$\bar{\mathbf{Q}}$	=	Rotated constitutive matrix for composite material
$Q_{ij}, \bar{Q}_{ij}$	=	components of constitutive matrix for composite material
$\mathbf{T}$	=	Transformation matrix
θ	=	Angle between the fibre orientation and the axis of the beam
EI	=	bending stiffness of column
L	=	length of column
$\mathbf{u}$	=	displacement vector
Π_p	=	potential energy functional
$\mathbf{C}$	=	elastic stiffness matrix
$\bar{\mathbf{F}}$	=	prescribed body force
$\bar{\mathbf{T}}$	=	boundary traction vectors
$\bar{\mathbf{u}}$	=	prescribed boundary displacements
$\mathbf{S}$	=	compliance matrix
$w(x)$	=	axial displacement of any point on the cross-section
$u(x)$	=	lateral displacement of any point on the cross-section
$v(x)$	=	vertical displacement of any point on the cross-section

$\phi(x)$	=	angle of twist of the cross-section
ε_x	=	axial strain
γ_{xz}	=	shear strain
$\bar{Q}_{ij}^{*}$	=	components of constitutive matrix in plane stress condition
$\hat{\bar{Q}}_{11}^{(k)*}$	=	components of constitutive matrix for with $\tau_{xy}^{(k)}=0$ assumption
$E_1^{(k)}, E_2^{(k)}$	=	Young's moduli of the k^{th} layer in two directions for orthotropic material
$\upsilon_{12}^{(k)}, \upsilon_{21}^{(k)}$	=	Poisson's ratios in two directions for orthotropic material
M	=	bending moment stress resultant
V	=	shear stress resultant
σ_x	=	normal stress
τ_{xz}	=	shear stress
J_{yy}	=	composite section constant
F_s	=	shear force
N^p	=	axial load at pre-buckling state
V^p	=	vertical load at pre-buckling state

M^p	=	bending moment at pre-buckling state
Π	=	total potential energy
U	=	strain energy
W	=	work done by external forces
V_0	=	volume of the element
β_1, β_2	=	Lagrange Multipliers
U_d	=	internal strain energy density
Π_{III}	=	hybrid functional
$\mathbf{L}^{\mathrm{T}}$	=	linear interpolation vector
$\mathbf{N}^{\mathbf{T}}$	=	cubic interpolation vector
$\mathbf{K}_{bi}$	=	element stiffness matrix
$\mathbf{K}_{gi}$	=	element geometric stiffness matrix
P_e	=	Euler buckling load
$\hat{N}_{\mathrm{crz}}$	=	buckling load of the column
$\hat{N}_{\mathrm{crz}}^{\mathrm{B}}$	=	buckling load when the shear stiffness is infinite
$\hat{S}_{\mathrm{yy}}$	=	buckling load when the bending stiffness is infinite

$w(s,z)$ = axial displacement of an arbitrary point $A(x,y)$

$u(s,z)$ = lateral displacement of an arbitrary point $A(x,y)$

$v(s,z)$ = vertical displacement of an arbitrary point $A(x,y)$

$W(z)$ = axial displacement of pole $P(a_x,a_y)$

$U(z)$ = lateral displacement of pole $P(a_x,a_y)$

$V(z)$ = vertical displacement of pole $P(a_x,a_y)$

ω = sectorial area

$\boldsymbol{\sigma}$ = stress vector

$\boldsymbol{\varepsilon}$ = strain vector

N = axial load

V_y = shear force

V_x = shear force

T_{sv} = St. Venant torsion

T_ω = twisting moment

τ_{zx} = shear stress

W_{Wagner} = Wagner stress resultant

I_p	=	sectional property
I_{py}	=	sectional property
I_{px}	=	sectional property
$I_{p\omega}$	=	sectional property
J_d	=	torsional constant
w_{sh}	=	vertical displacement in the shell element
u_{sh}	=	displacement in x direction of the shell element
w_{sh}	=	displacement in y direction of the shell element
$\theta_{x,sh}$	=	bending rotation about x axis in the shell element
$\theta_{y,sh}$	=	bending rotation about y axis in the shell element
$\theta_{z,sh}$	=	bending rotation about z axis in the shell element

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Figure 7.14: Cross-sectional dimensions of the channel-section

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Abstract

Thin-walled members are widely used in mechanical and civil engineering applications. The use of thin-walled elements made of fibre-reinforced composite materials has increased significantly in the past decades due to the superior features of these materials. However, because of their slenderness, susceptibility of thin-walled composite members to buckling is the main concern in the structural design of these elements. For the buckling analysis of thin-walled members with any loading types and boundary conditions, one tends to use numerical methods rather than the closed-form solutions which are limited to simple loading and boundary conditions. Finite element methods (FEM) as the most commonly used numerical techniques can be categorised into two main groups: single-field FEM and multi-field or hybrid FEM. The first group is further categorised into two types: displacement-based elements and stress-based elements.

In buckling analysis of thin-walled members with fibre-reinforced laminated composite materials, shear deformations can have a significant effect. Single-field finite element methods adopt different approaches to include shear deformations. Displacement-based methods take account of the effects of shear deformations by modifying the kinematic assumptions of the thin-walled theory. On the other hand, in stress-based methods, the inter-element equilibrium conditions have to be satisfied a-priori, which further complicates the assemblage procedure.

A shear-deformable hybrid finite element method for the buckling analysis of composite thin-walled members is developed in this thesis by enforcing the strain-displacement relations to the potential energy functional. In the developed method, the resulting matrix equations are defined only in terms of the nodal displacement values as

unknowns which makes the assemblage procedure as simple as in a displacement-based finite element. The shear deformations are taken into account in the current hybrid finite element method by using the strain energy of the shear stress field which eliminates the mentioned difficulties in the other finite element methods.